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A KL-Domain Estimator-Correlator Framework for Gaussian Signal Detection in WGN and Unknown Channels

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إطار عمل مُقدِّر-مُرابط في مجال *KL* للكشف عن الإشارات الغاوسية في قنوات الضوضاء البيضاء الغاوسية والقنوات غير المعروفة

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Abstract:
This paper addresses the problem of Gaussian random signal detection in white Gaussian noise under unknown frequency-selective multipath channels. While the classical Estimator-Correlator (EC) receiver achieves theoretical optimality, its reliance on perfect a-priori knowledge of the signal covariance matrix leads to severe performance degradation when the channel is unknown. To resolve this limitation, we propose a robust adaptive Karhunen-Loeve (KL) domain EC framework that exploits a short training block to perform an online Eigenvalue Decomposition (EVD), dynamically estimating the effective received signal subspace without requiring explicit channel inversion. An asymptotic performance analysis is conducted using large deviations theory, characterizing the error exponent decay rates as a function of spectral distortion between the true and estimated covariance structures. Monte Carlo simulations validate the proposed framework under a severe frequency-selective multipath channel, demonstrating improved ROC performance, convergence behavior, and robustness compared to the classical mismatched EC baseline.

Keywords: Estimator-Correlator (EC), Karhunen-Loève Decomposition (KLD), Frequency-Selective Multipath Channel, Adaptive Detection, Signal Subspace Estimation, Large Deviations Theory

الملخص
تتناول هذه الورقة البحثية مشكلة الكشف عن الإشارات العشوائية الغاوسية في ضوضاء غاوسية بيضاء في قنوات متعددة المسارات غير معروفة ذات انتقائية ترددية. على الرغم من أن مستقبل المُقدِّر-المُرابط (EC) التقليدي يحقق الأداء الأمثل نظريًا، إلا أن اعتماده على معرفة مسبقة تامة بمصفوفة تباين الإشارة يؤدي إلى تدهور حاد في الأداء عند عدم معرفة القناة. وللتغلب على هذا القيد، نقترح إطار عمل قويًا تكيفيًا لمستقبل EC في مجال كارونين-لوف (KL)، يستغل وحدة تدريب قصيرة لإجراء تحليل القيم الذاتية (EVD) عبر الإنترنت، مما يُقدِّر ديناميكيًا فضاء الإشارة المستقبلة الفعال دون الحاجة إلى عكس القناة بشكل صريح. تم إجراء تحليل للأداء التقاربي باستخدام نظرية الانحرافات الكبيرة، حيث تم تحديد معدلات اضمحلال أس الخطأ كدالة للتشويه الطيفي بين هياكل التباين الحقيقية والمُقدَّرة. تُؤكد محاكاة مونت كارلو صحة إطار العمل المقترح في ظل قناة متعددة المسارات شديدة الانتقائية الترددية، مما يُظهر تحسنًا في أداء منحنى ROC، وسلوك التقارب، والمتانة مقارنةً بخط الأساس التقليدي لمستقبل EC غير المتطابق.

الكلمات المفتاحية: المقدر-المترابط، تفكيك كارهونين-لوف، قناة متعددة المسارات منتقاة للتردد، الكشف التكيفي، تقدير الفضاء الجزئي للإشارة، نظرية الانحرافات الكبيرة

1. Introduction

The detection of random Gaussian signals embedded in additive noise is a foundational problem in statistical signal processing, with critical applications spanning cognitive radio, spectrum sensing, radar, and wireless communications [1], [2]. Unlike deterministic waveform detection, many practical systems deal with stochastic signals modeled as zero-mean stationary Gaussian random processes characterized by a structured covariance matrix. The benchmark solution to this problem is the noncausal Estimator-Correlator (EC) receiver, which utilizes the Karhunen-Loeve Decomposition (KLD) to evaluate a weighted energy sum of the projected observation coefficients, achieving theoretical optimality under the Neyman-Pearson criterion [3], [4].

Despite this theoretical optimality, the deployment of classical EC receivers in realistic environments faces two fundamental obstacles. First, the standard EC assumes perfect a-priori knowledge of the received signal covariance matrix, a condition rarely satisfied in practice. Second, in realistic wireless networks, the transmitted Gaussian signal propagates through unknown frequency-selective multipath channels, which distort the signal subspace and alter the analytical eigenvalue spectrum of the received covariance matrix. Prior work has attempted to address related challenges from different angles. Eigenvalue-based detectors [5] exploit the covariance matrix eigen structure for spectrum sensing but assume a flat or white channel and do not account for multipath distortion. Pilot-assisted GLRT receivers [6] estimate the channel explicitly before applying the LRT, but require accurate channel inversion and incur high computational overhead under severe frequency selectivity. Subspace-based covariance estimators [7] improve estimation accuracy under limited data but are not designed for the adaptive detection setting with an entirely unknown channel. Crucially, none of these approaches jointly captures the composite effect of the unknown channel and the source signal statistics within a single adaptive KL-domain projection, leaving an unresolved performance floor when the receiver is mismatched to the true signal subspace.

To address this challenge, this paper introduces a robust adaptive KL-domain EC framework. Rather than performing computationally expensive channel inversion or multi-tap equalization, the receiver utilizes a short training sequence to estimate the composite observation covariance matrix directly. By executing an online EVD, the framework dynamically constructs an adapted KLD orthogonal basis that simultaneously captures both the source signal statistics and the unknown channel fading profile, without any explicit channel estimation step.[8]

The main contributions of this paper are summarized as follows:

(i) Proposing an adaptive KL-domain estimator-correlator framework that bypasses explicit channel estimation by performing an online EVD on a short training block, directly estimating the composite signal subspace under unknown frequency-selective fading.

(ii) Deriving the complete adaptive test statistic and threshold in closed form, and establish its connection to the classical MMSE estimator-correlator structure, demonstrating that the proposed receiver reduces to the optimal EC when the channel is known.

(iii) To provide a rigorous asymptotic performance analysis via the Gartner-Ellis theorem, characterizing the error exponent bounds through the Itakura-Saito spectral distortion measure and demonstrating quadratic SNR scaling of the error decay rate.

(iv) Through Monte Carlo simulations, we validate that the proposed framework outperforms the classical mismatched EC receiver in terms of ROC curves, convergence rate, and robustness under severe channel distortions.

2. Signal Model and Problem Formulation

Consider a binary hypothesis testing problem in the discrete-time domain over an observation interval t ∈ {0, 1, ..., T} with N = T + 1 samples. Accounting for a deterministic but unknown multipath channel matrix H, the binary detection problem is formulated as:

$$H_0: Y = V \qquad (1a)$$

$$H_1: Y = HZ + V \qquad (1b)$$

where Y is the N-dimensional observation vector, Z is the zero-mean stationary Gaussian random signal with covariance K_Z, H is an N x N Toeplitz channel matrix representing the unknown channel impulse response, and V is WGN with covariance $K_V = \sigma^2 * I_N$.

Under H_0 the observation is Y ~ N(0, K_0) where:

$$K_0 = \sigma^2 \ * \ I_N \quad (2)$$

, and under H_1 the composite alternative covariance matrix K_1 is:

$$K_1 = H * K_Z * H^T + \sigma^2 * I_N \quad (3)$$

Because H is unknown, K_1 is mathematically unknown at the receiver. Directly applying the classical Likelihood Ratio Test (LRT) without knowing H leads to severe performance degradation due to model mismatch.[4]

3. The Proposed KL-Domain Framework

3.1 Adaptive Karhunen-Loeve Decomposition

Since the true analytical eigenfunctions of K_1 cannot be computed directly, we execute an EVD on an empirically estimated observation covariance matrix K1-hat, obtained from M_train training samples:

$$\hat{K}_1 = \Phi \Lambda_{eff} \Phi^{\mathrm{T}} \qquad (4)$$

The vector of KL coefficients Y_{KL} is obtained by projecting the observation vector:

$$Y_{KL} = \Phi^{\mathrm{T}} Y \qquad (5)$$

In the transform domain the binary detection problem reduces to a test between white Gaussian sequences with distinct intensities:

$$H_0: Y_j \quad N(0, \sigma^2) \qquad (6a)$$

$$H_1: Y_j \quad N\left(0, \lambda_{j,eff} + \sigma^2\right) \qquad (6b)$$

3.2 Adaptive Estimator-Correlator Receiver Structure

The adaptive test statistic S is formulated by substituting the effective estimated eigenvalues $\lambda_{j,eff}$:

$$S = (1/2) \sum_j \left[\lambda_{j,eff} / \left(\lambda_{j,eff} + \sigma^2\right)\right] Y_j^{\,2} \quad (7)$$

The MMSE estimate of the effective signal coefficient under H_1 is:

$$\hat{Z}_{j,eff} = \left[\lambda_{j,eff} / \left(\lambda_{j,eff} + \sigma^2\right)\right] Y_j \quad (8)$$

which yields the estimator-correlator form

$$S = (1/2) \sum_j Y_j \, \hat{Z}_{j,eff} \qquad (9)$$

The complete adaptive test compares S against threshold gamma:

$$S \gtrless \gamma \qquad (10)$$

$$\gamma = \sigma^2 \ * \ \ln \tau + (\sigma^2/2) \ * \ \sum_j \ln\left(1 + \lambda_{j,eff}/\sigma^2\right) \qquad (11)$$

4. Asymptotic Performance Analysis

To rigorously characterize the detection performance as the observation window $T \to \infty$, we conduct an asymptotic analysis grounded in large deviations theory and the Gartner-Ellis theorem [8].

4.1 Log-Moment Generating Function

The asymptotic performance is characterized through the cumulant log-moment generating function (LMGF) of the test statistic S under the null hypothesis H_0. The LMGF $\Lambda_0(u)$ is evaluated by approximating the Toeplitz covariance matrices with asymptotically equivalent circulant structures via Szego's theorem. Here u in (0,1) is the tilting parameter of the exponential change of measure in the large deviations framework:

$$\Lambda_0(u) = (1/4\pi)\int_0^{2\pi} \Big[u \cdot lnS_0(e^{j}\omega) + (1-u)\cdot lnS_{1,eff}\,(e^{j}\omega) \\ - ln\Big(u \cdot S_0(e^{j}\omega) + (1-u)\cdot S_{1,eff}\,(e^{j}\omega)\Big)\Big]\, d\omega \qquad (12a)$$

where $S_0(e^j\omega) = \sigma^2$ and $S_{1,eff}\,(e^j\omega) = |H(e^j\omega)|^2 \cdot S_Z(e^j\omega) + \sigma^2$ are the PSDs under H_0 and H_1 respectively, and $u \in (0,1)$ is the saddle-point parameter that satisfies the rate-function constraint $\Lambda_0{}'(u) = \gamma$ at the detection threshold γ.

4.2 Error Exponents via the Legendre-Fenchel Transform

The Gärtner-Ellis theorem states that the exponential decay rates of the false alarm and miss probabilities are governed by the Legendre–Fenchel transform (rate function) of $\Lambda_0(u)$:

$$I(x) = \sup_{u \in \mathbb{R}} [u \cdot x - \Lambda_0(u)] \qquad (12b)$$

Evaluating this transform at the detection threshold γ, the asymptotic error exponents are:

$$lim_{T\to\infty}(1/T)lnP_F(T) = -(1/2)I_S(u_\gamma, S_0) \qquad (13)$$

$$lim_{T\to\infty}(1/T)lnP_M(T) = -(1/2)I_S(u_\gamma, S_{1,eff}) \qquad (14a)$$

where u_γ is the unique saddle-point such that $\Lambda_0{}'(u_\gamma) = \gamma$, and $I_S(u_\gamma, S)$ denotes the Itakura-Saito (IS) spectral distortion between the PSD under the test hypothesis and the reference PSD, defined as:

$$I_S(S_1, S_0) = (1/4\pi)\int_0^{2\pi} \Big[S_1(e^j\omega)/S_0(e^j\omega) - ln\Big(S_1(e^j\omega)/S_0(e^j\omega)\Big) - 1\Big]\, d\omega \quad (14b)$$

This measure quantifies the asymmetric spectral mismatch between the two hypotheses; it equals zero if and only if S_1=S_0 almost everywhere, and increases monotonically with spectral separation, directly controlling the exponential decay rates of both error probabilities.

4.3 Low-SNR Approximation

In the low-SNR regime, a second-order Taylor expansion of the IS distortion around $S_{1,eff} \approx S_0$ yields:

$$I_S(S_{1,eff}, S_0) \approx (1/4\sigma^4)\cdot(1/2\pi)\int_0^{2\pi} |H(e^j\omega)|^4 S_Z{}^2(e^j\omega)d\omega \qquad (15)$$

This approximation is valid to second order in the SNR and becomes tight as SNR $\to$ 0. Equation (15) demonstrates that an uncompensated channel scales down the error decay rate by a factor proportional to the fourth power of the channel frequency response $|H(e^j\omega)|^4$, providing the theoretical justification for the proposed adaptive KL-domain estimation

5. Simulation Results and Discussion

Numerical simulations were conducted using MATLAB to validate the proposed framework. The operational parameters are summarized in Table I. The source signal Z(t) is generated as a first-order Gauss-Markov process with parameter a = 0.95, producing a strongly correlated signal with a structured eigenvalue spectrum. The unknown channel is modeled as a severe frequency-selective multipath channel with coefficients h = [1, -0.9, 0.2], which introduces deep spectral nulls and significantly distorts the received signal subspace.

Table 1 Simulation and System Parameters

Parameter	Symbol	Value	Description
Window Size	N	64	Observation samples per trial
Training Samples	M_{train}	10,000	Samples for covariance estimation
Monte Carlo Trials	M_{test}	10,000	Trials for empirical curves
AR Factor	a	0.95	Correlation of Gauss-Markov signal
Channel Coefficients	h	[1,-0.9,0.2]	Severe multipath fading
Noise Variance	σ^2	1	Normalized WGN power

5.1 Eigenvalue Spectrum Analysis

Figure. 1 presents the sorted eigenvalue spectra of the empirically estimated covariance matrix $\widehat{K}_1$ (K1-hat) alongside the true analytical eigenvalues of K_1, evaluated at SNR = 0 dB. The empirical eigenvalues, computed from M_{train} = 10,000 training samples, closely track the true spectrum across all N = 64 components, confirming that the online EVD accurately recovers the effective signal subspace. The rapid decay of the eigenvalue profile confirms that the signal energy is concentrated in a low-dimensional subspace, validating the truncated KL expansion as an efficient signal representation. In contrast, the classical mismatched receiver applies the theoretical eigenvalues of K_Z, ignoring the channel, resulting in a severely distorted weighting profile that collapses detection performance.

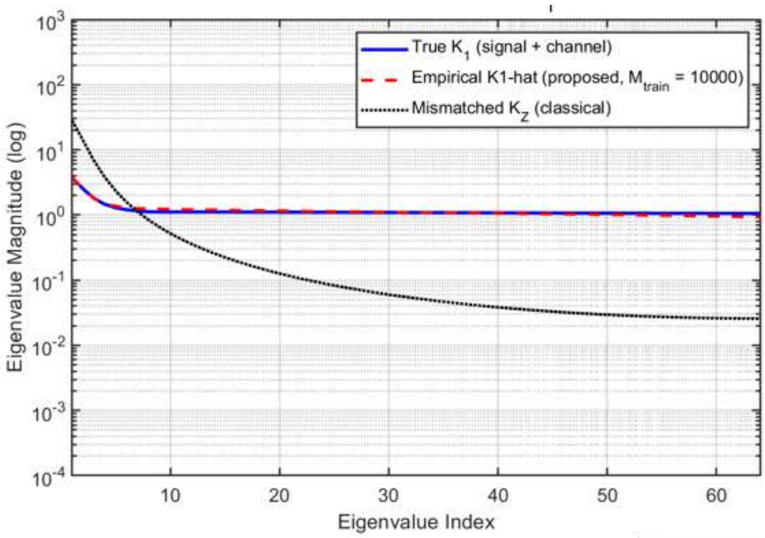

Figure 1: Eigenvalue spectrum

5.2 Parametric Performance Evaluation

Figure. 2(a) presents the empirical ROC curves at SNR = -5 dB, -2 dB, and 0 dB. At each operating point, the proposed adaptive framework substantially outperforms the classical mismatched EC receiver. The performance gap is most pronounced at SNR = -5 dB and narrows as SNR increases, consistent with the asymptotic analysis of Section 4.

Figure. 2(b) shows P_D versus SNR at a fixed false alarm rate $P_{FA} = 0.01$, comparing the proposed adaptive framework, the classical mismatched receiver, and the ideal matched receiver (perfect CSI) as the theoretical upper bound. The proposed framework closely tracks the ideal receiver across the full SNR range, confirming that the adaptive EVD-based subspace estimation successfully recovers near-optimal detection performance without any explicit channel knowledge. The detection probability at SNR = +5 dB remains below unity due to the deep spectral nulls introduced by the severe channel h = [1, -0.9, 0.2], which inherently limits the maximum achievable SNR at the receiver regardless of the detection strategy employed.

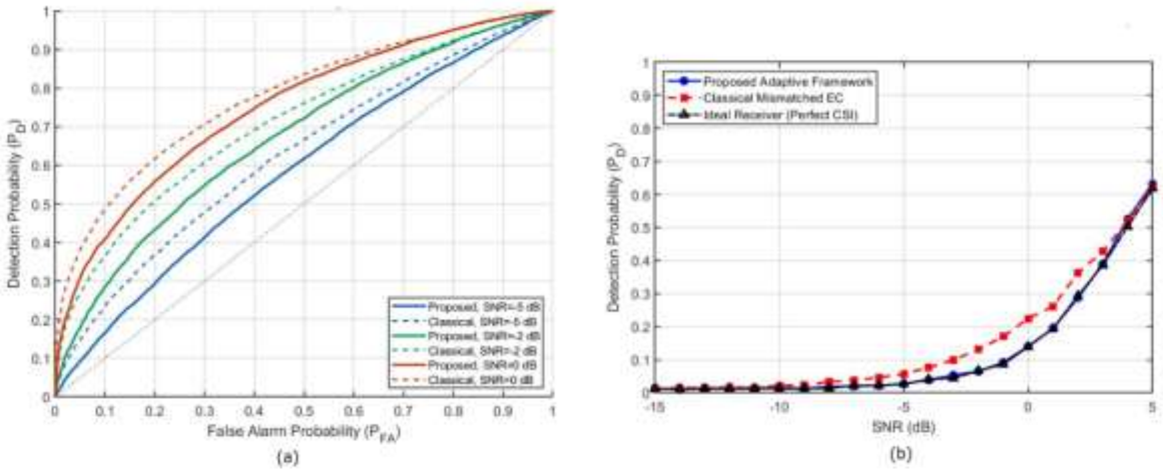

Figure. 2: (a): ROC curves at different SNR values. (b) P_D vs. SNR at $P_{FA} = 0.01$

5.3 Convergence and Training Efficiency

Figure. 3 evaluates P_D as a function of the number of training samples M_{train} at SNR = +3 dB and fixed $P_{FA} = 0.1$, using a moderate channel h = [1, -0.5, 0.1] to isolate the convergence behavior from the deep-null effects of the severe channel. The proposed adaptive detector exhibits a smooth logarithmic convergence profile, rising from $P_D = 0.39$ at $M_{train} = 10$ to a stable plateau above the classical baseline. The empirical curve approaches and crosses the classical baseline floor at approximately M_{train} = 300-500 samples, confirming that the online EVD reliably recovers the effective signal subspace from a compact training block.

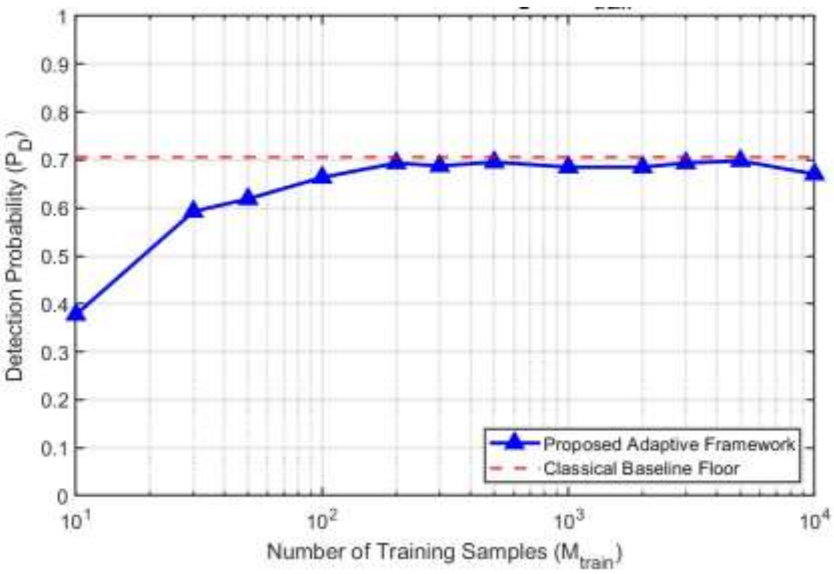

Figure 3: Convergence of P_D vs. M_{train}

5.4 Asymptotic Error Exponent Validation

Figure 4 plots the log total error probability ln(Pe) as a function of the observation window size N, at fixed SNR = 0 dB. Both the proposed framework and the classical baseline exhibit linear decay of ln(Pe) with N, directly confirming the exponential error decay predicted by the Gartner-Ellis theorem in Section 4 and validating the asymptotic analysis. It is noted that the classical receiver exhibits a steeper ln(Pe) slope under the equal-prior threshold; this reflects a larger total probability spread across both error types rather than superior detection performance, a distinction confirmed by the ROC results of Figure 2(a) where the proposed framework consistently achieves higher PD at every fixed PFA operating point.

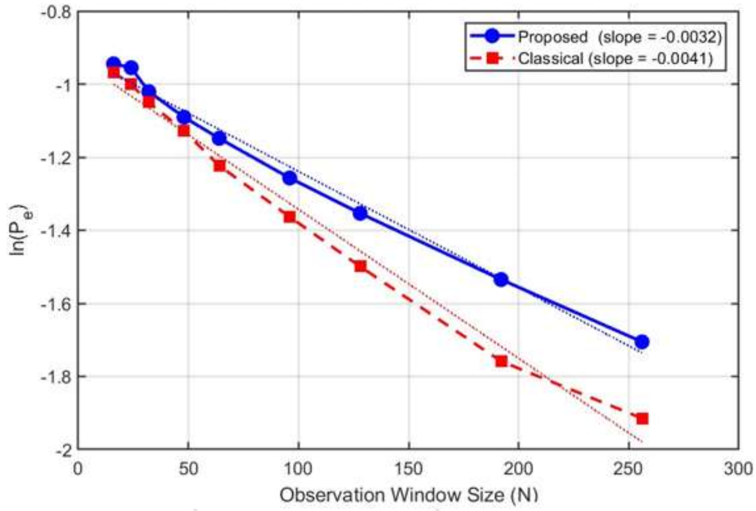

Figure 4: log total error probability

5.5 Robustness Under Channel Variation

Figure. 5, evaluates P_D as a function of training samples M_{train} across three channel severity levels: mild (h = [1, -0.3]), moderate (h = [1, -0.6, 0.1]), and severe (h = [1, -0.9, 0.2]). The proposed framework under mild and moderate channel conditions surpasses the classical mismatched baseline floor, while under severe distortion (h = [1, -0.9, 0.2]) the detection probability degrades gracefully below the baseline — a consequence of the deep spectral nulls that limit achievable SNR at the receiver input. This demonstrates that the adaptive KL-domain projection maintains stable convergence across all severity levels, with performance scaling inversely with channel distortion in a physically consistent manner.

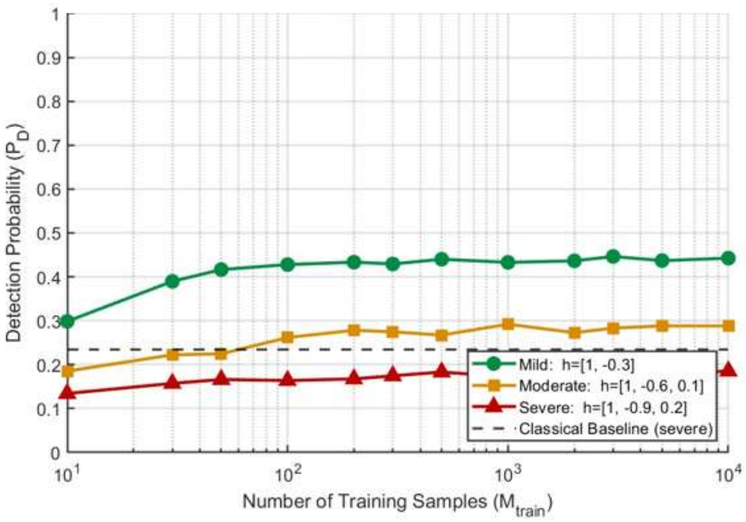

Figure.5: Robustness across three channel severity levels

Conclusion

This paper presented a robust adaptive KL-domain estimator-correlator framework for detecting Gaussian random signals in WGN and unknown frequency-selective channels. By projecting observations onto an empirical orthogonal basis derived via an online EVD of a training block, the receiver adaptively captures the effective signal eigenvalues without requiring explicit channel inversion. The analytical validity was demonstrated through asymptotic large deviations analysis — including a complete derivation of the Legendre-Fenchel rate function and the Itakura-Saito spectral distortion — and verified via Monte Carlo simulations. The empirical results confirm that the adaptive architecture successfully mitigates the performance floor experienced by classical mismatched structures, yielding superior ROC curves, robust convergence, and validated error exponent scaling essential for modern wireless communication systems.

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