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Analytical Representation of Lightning Current Impulses

Saad Khalifa Daw [1] , Sadek M. F. Elkurie[2], Abdulwahid. A. khalleefah [3]
[1,2]Department Electric and Electronic, College of Engineering, Nalut University, jado, Libya
[3]Higher Institute of Science and Technology Alasabaa, Alasabaa, Libya

التمثيل التحليلي لنبضات تيار البرق

سعد خليفة ضوء [1]*، الصادق محمد الكري[2]، عبدالواحد ابراهيم خليفة [3]
[1،2] قسم الهندسة الكهربائية والكترونية، كلية الهندسة، جامعة نالوت، جادو، ليبيا
[3] قسم الهندسة الكهربائية، المعهد العالي للعلوم والتقنية ، الاصابعة ، ليبيا

*Corresponding author: S.khelifaomardaw@nu.edu.ly

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Abstract:
Lightning is a natural phenomenon that generates high-magnitude current impulses, posing significant risks to electrical and structural systems. The accurate modeling of lightning current waveforms is crucial for designing effective protection systems. This paper presents an analytical representation of the lightning current impulse, focusing on its waveform characteristics, key parameters, and mathematical modeling. Various methods for approximating the current front and tail are discussed, along with simulations that illustrate the behavior of lightning strokes. The study provides insights into the analytical techniques used to model lightning surges, aiding engineers in improving lightning protection strategies.

Keywords: lightning current impulse, analytical representation protection, mathematical modeling, waveform analysis.

الملخص
يعد البرق ظاهرة طبيعية تولد نبضات تيار عالية الحجم، مما يشكل مخاطر كبيرة على الأنظمة الكهربائية والإنشائية. إن النمذجة الدقيقة لأشكال موجات تيار البرق أمر بالغ الأهمية لتصميم أنظمة حماية فعالة. تقدم هذه الورقة البحثية تمثيلاً تحليلياً لنبضات تيار البرق، مع التركيز على خصائص شكل موجته ومعلماته الرئيسية ونموذجه الرياضي. وتناقش طرقًا مختلفة لتقريب مقدمة التيار ونهايته، إلى جانب عمليات المحاكاة التي توضح سلوك ضربات البرق. تقدم الدراسة نظرة ثاقبة على التقنيات التحليلية المستخدمة في نمذجة اندفاعات الصواعق، مما يساعد المهندسين في تحسين استراتيجيات الحماية من الصواعق.

الكلمات المفتاحية: نبضة تيار البرق، الحماية بالتمثيل التحليلي، النمذجة الرياضية، تحليل شكل الموجة.

Introduction

Accurate knowledge of lightning current parameters is essential for the appropriate analysis risk of overhead electric devices such as overhead transmission lines, power stations or buildings during lightning. The properties of lightning have a statistical character thus, some measured values are needed to determine their statistical distribution. However, to collect sufficient data and measurements must be made on high objects where a high frequency of lightning strikes may be expected. Therefore, even commonly known lightning parameters are related to high object. An important value is ratio of positive or negative lightning flashes to total lightning flashes. According to the observations high objects are struck by negative lightning with 90 % or higher probability. This ratio seems to be generally valid, independent of the height of the objects. It is possible that the ratio of positive strikes is a little higher at high voltage transmission lines than at towers [1,2,3]. In order for society to properly plan this critical infrastructure to achieve the desired growth rate[4].

Material and methods.

I.Lightning Current Wave shape

The wave shape of lightning currents have been analyzed in many publications [1,3, 5,6,7]. It has very important influence on time courses and maximal values of generated overvoltages. The following basic parameters should be known to determination of lightning current:

- current peak value,
- maximum of the current steepness
- the charge transfer at the striking point.

Typical course of lightning currents of negative polarity is presented in Figure 1.

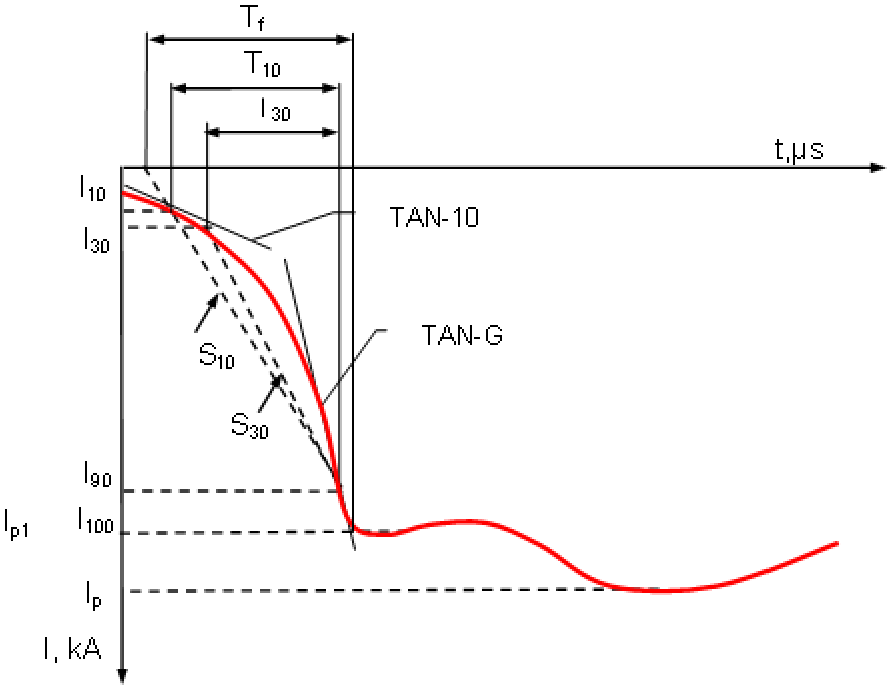

Figure 1. Definition of front parameters for a lightning current impulse of negative polarity:

I_{p1} - maximal value of first current peak, A,

I_p - maximal value of highest current peak, A,

T_{10} - time interval between instants corresponding to 10 % and 90 % of first peak value, s,

T_d - front duration according to T_{10}, s

T_{30} - time interval between instants corresponding to 30 % and 90 % of first peak value, s,

T_{d30} - front duration according to T_{30} ($T_{30} \cong T_{d30}/0.6$), s,

S_{10} - average front steepness between 10 % and 90 % value point of first peak amplitude, A s-1,

S_{30} - average front steepness between 30 % and 90 % value point of first peak amplitude, A s-1,

TAN-10 - rate of current rise at to 10 % value point of first peak amplitude,

TAN-G - the maximum front steepness or rate of rise in the front wave,

T_f - front duration ($T_f = 1.25\ T_{10}$)

Note that the statistical parameters based on the 10 % intercept will consistently be less reliable than the 30 % values, due to the use of a 2 kA reference level for measurements. For engineering purposes therefore, the 30 % based parameters should generally be used, as indicated in the following sections. The frequency distribution of these additional impulse front parameters is summarized in Table 2.3, assuming a log-normal distribution of variables, where the general equation for the probability density for any particular parameter x is given by [5,8,9].

$$\mathrm{f(x)} = \frac{1}{\sqrt{2\pi}\beta x} e^{-\frac{z^2}{2}} \quad (1)$$

where:

$$z = \frac{\ln\left(x/_M\right)}{\beta} \quad (2)$$

M the median parameter value,

β - the slope parameter or logarithmic standard deviation (base e).

The mean value of any parameter may then be expression as:

$$\mu = Me^{\frac{\beta^2}{2}} \quad (3)$$

2 Parameters of Lightning Strokes

Lightning being assumed to be a current source; the magnitude and shape of the return-stroke current wave play a significant role in the estimation of outage rates of power systems caused by lightning. The return-stroke current rises to its peak in a few microseconds and slowly decays after reaching the peak. The time to peak is called the front time, t_f, and the time duration from $t = 0$ to the instant when the current subsequently decays to the 50 % value of the peak is called the time to half value (tail time) t_h. The time to half value, t_h, being many times longer than t_f , does not play a significant role in the severity of lightning-caused transient over voltages. However, the influence of the peak of the current wave Ip and t_f is very significant [1, 3,9].

Table 1. Parameters of log-normal distribution for negative downward flashes

Parameter	First Stroke		Subsequent Stroke	
	M	β	M	β
front, µs				
$t_{d10} = T_{10}/0.8$	5.63	0.576	0.75	0.921
$t_{d30} = T_{10}/0.6$	3.83	0.553	0.67	1.013
steepness, kA µs $^{-1}$				
S_m, maximum	24.3	0.599	39.9	0.852
S_{10}, at 10 %	2.6	0.921	18.9	1.404
$S_{10/90}$, 10 - 90 %	5.0	0.645	15.4	0.944
$S_{30/90}$, 30 - 90 %	7.2	0.622	20.1	0.967
crest current, kA				
I_I, initial	27.7	0.461	11.8	0.530
I_F, final	31.1	0.484	12.3	0.530
initial/final	0.9	0.230	0.9	0.207
tail, t_h, µs	77.5	0.577	30.2	0.933
charge, Q_I, C	4.65	0.882	0.938	0.882
$\int i^2\, dt$, $(\mathrm{kA})^2\mathrm{s}$	0.057	1.373	0.0055	1.366
inter stroke interval, ms	-	-	35	1.066

3 Characteristic and Parameters of Lightning Strokes

The lightning flash is categorized by the polarity of the cloud and the direction of propagation of the flash leader. Hence, there are four categories of lightning flash to ground in which the developed leader is followed by a return-stroke current impulse, namely [1].

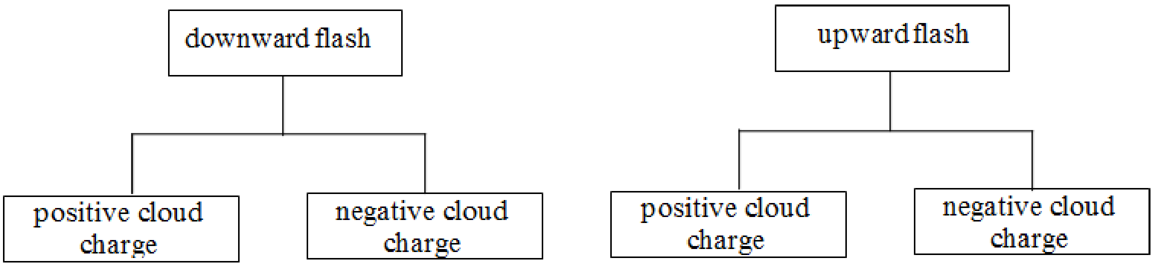

Figure 2. Categories of lightning flash to ground

On average, at least 90 % of downward flashes are of negative polarity, with some 45 - 55 % of flashes comprising only one stroke. Multiple stroke flashes seldom involve more than 10 strokes (less than 5 %), and generally average three strokes per flash, typically at intervals of less than three strokes per flash, per stroke. Upward flashes occur mainly very tall towers. The majority of transmission line structures are of only moderate height (typically less than 60-100 m) and will not in general be subject to upward flashes. The lightning flash many contain several lightning strokes whereby the first steeped-leader/return-stroke sequence is followed in shortly succession by a series of one or more subsequent strokes. Each stroke comprises a dart-leader and return-stroke sequent that generally follows the breakdown path of the first-stroke. Lightning protection systems for transmission line must therefore be capable of withstanding the effects of a series of lightning strokes to the same location within a short period of time. Each lightning stroke is considered an ideal current source of infinite source impedance and the parameter contained within the incident impulse current wave shape determine the incident impulse current wave shape determine the transmission network response. This wave shape parameter include the peak-current (crest value), time to crest, steepness and duration. Also of importance are the polarity, time interval and number of incident strokes within each lightning flash. Basic parameters values of lightning discharge be subject to distribution variation. Variety type lightning discharge charge in cloud and atmospheric geographic to cause that the plot a curve statistical distribution value of the above parameters. Lightning discharge parameters had characteristic randomly. Probability occurrence of peak values of lightning current is presented in Figure 3 [1, 7,10].

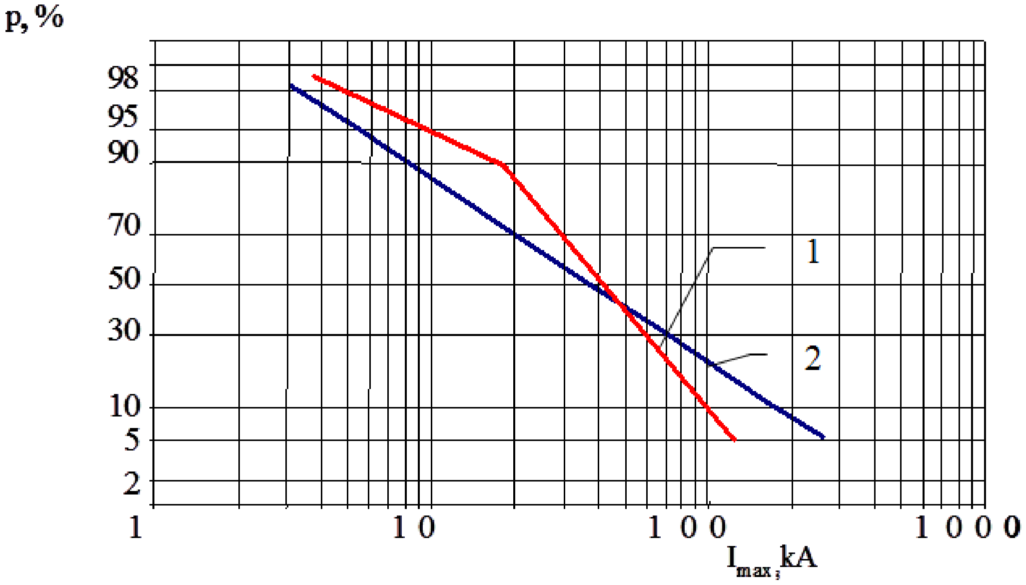

Figure 3. Distribution functions of the peak value of lightning discharge: 1 - the first negative stroke,

2 - the first positive stroke

As in the result in Figure 2.3 high probability values from lightning that increase values of parameter of lightning discharge, they had significant discharge induced overvoltage in electrical system conductivity and power supply.

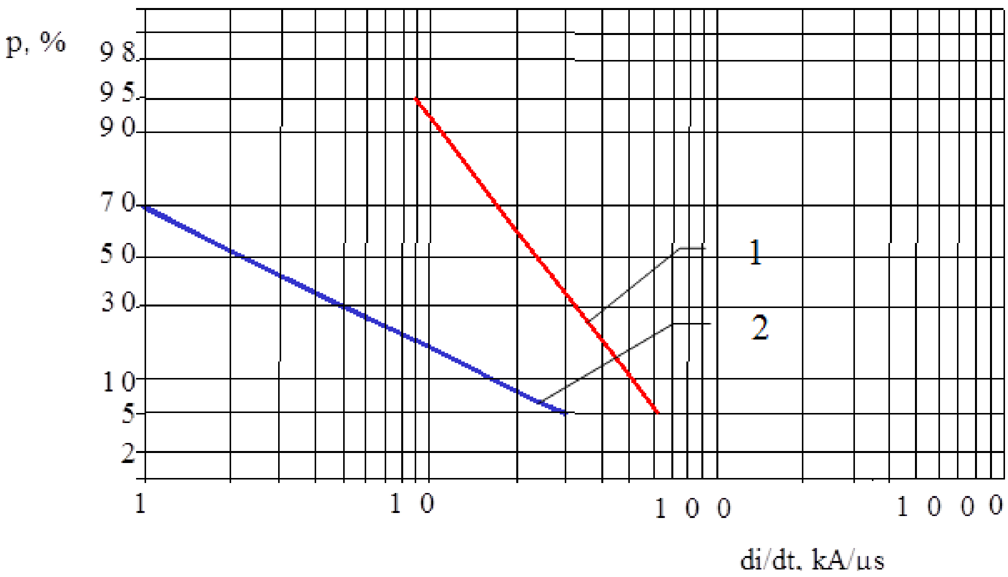

Figure 4. Distribution of rate of rise *di/dt*: 1 - the first negative stroke, 2 - the first positive stroke

There are some parameters which characterize the effects causing damage. Overvoltage induced by the lightning channel or the current flowing in a conductor is proportional to the rate of rise in the lightning current. Many semiconductor devices can be damaged by a very short overvoltage, therefore, the maximum *di/dt* determines the hazard. A longer time duration (at least 1 µs) is needed to produce a discharge in a airgap or breakdown of solid insulating material and so the hazard on an average rate during the front 1of the current wave. The latter value is related to the part of the front rising from 10% up to 90% of the peak value. Where the lightning cannel is in contact with a metal object the melting effect depends on the charge flowing through the spot of contact.

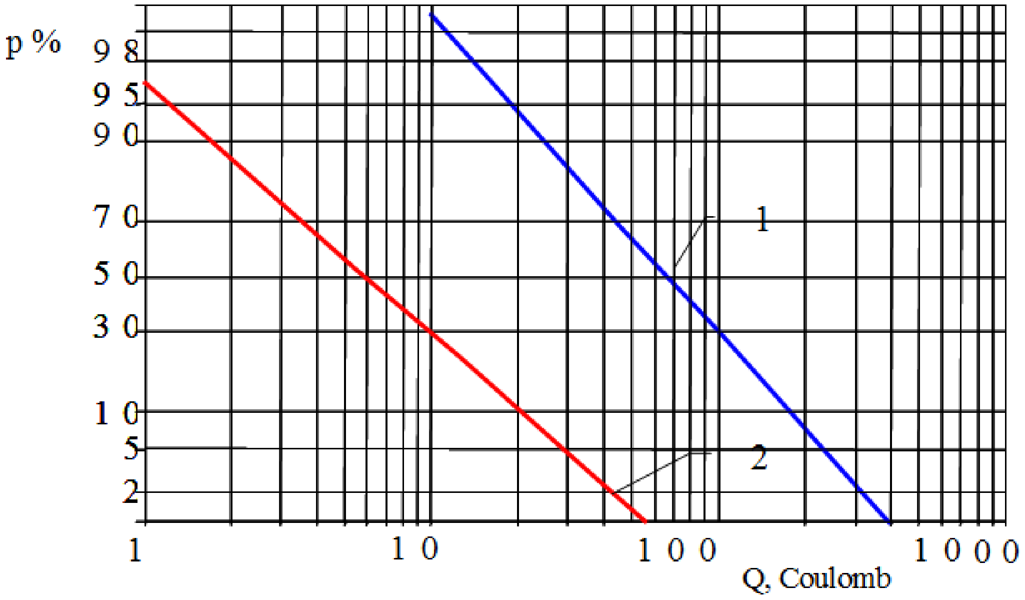

Figure 5. Distribution of charge of lightning: 1- positive stroke, 2- negative stroke

Figure 5, shows the distribution diagram of the charge related to the total flash and to the current impulse. It can be seen that the highest values occur at the positive lightning strikes.

4Mathematical Modeling of Lightning Surges

A lightning flash can result in significant overvoltage on power line insulation if substantial stroke current and charge are injected into the line conductor. If first contact of lightning return stroke current is with the line, then it is modeled as a transient current generator feeding into a system of transient surge impedance representing the line conductors and the tower, and then the overvoltage's results are calculated using typical traveling wave technique.

The impedance of return stroke is assumed to be larger than the surge impedance of the line or tower, because of the value of the current initiated from the lightning stroke, then may be neglected. The return stroke can then be modeled as an ideal current generator having selected magnitude and wave shape.

The wave shape parameters include the peak current amplitude (crest value), time to crest, steepness, duration, polarity, time interval and number of incident strokes within each lightning flash. In earlier analysis, the front characteristics were defined in the term of the maximum rate of rise of current.

5 Analytical Representation of the Current Shape

To calculate the lightning performance of equipment, it is necessary to simulate the concave front in the lightning stroke current representation. Three points are needed to establish such simulation:

- the correct amplitude of the current,
- the highest steepness close to the peak amplitude,
- for first strokes, the correct average steepness expressed as the front time passing through the 30 % and the 90 % values of current. This front time must be larger than the current amplitude divided by the maximum steepness, thus resulting in the concave shape. For subsequent stroke this parameter may be neglected.

Many mathematical expressions may be suitable to fulfill these requirements and the one given here is only one proposal. Its disadvantage is that the current front and the current tail are not described by a single expression, but are separated into two parts, one describing the front up to 90 % of the maximal value, the other, the maximal value on the tail [11].

A. The Current Front

The current front for first strokes can be expressed as:

$$I = At + Bt^n \tag{4}$$

where: A, B - constants.

The basic assumption is that the current shape reaches the instant of maximum steepness (90 % amplitude) at a time t_n dependent on exponent n. in principle, both variables have to be evaluated by an iterative solution of the generalized equation:

$$\left(1-\frac{3x}{2S_N}\right)(1-x)^n = x\frac{n-1}{2S_N}+\frac{1-3xn}{2S_N}(1-x) \tag{5}$$

with:

$$S_N = S_m\,\frac{t_f}{I} \tag{6}$$

$$X = 0.6\,\frac{t_f}{t_n} \tag{7}$$

where: I - maximal value of current, A,

S_m - maximum steepness, $A\,s^{-1}$,

t_f - front time, s.

However, a sufficiently accurate is given by:

$$n = 1 + 2(S_N - 1)\left(2+\frac{1}{S_N}\right) \tag{8}$$

$$t_n = 0.6\,t_f\,\frac{3S_N^2}{1+S_N^2} \tag{9}$$

The constants then are:

$$A = \frac{1}{n-1}\left(0.9\frac{I}{t_n}n - S_m\right) \tag{10}$$

$$B = \frac{1}{t_n^n(n-1)}(S_m t_n - 0.9I) \tag{11}$$

For subsequent strokes the current front is given by:

$$I = S_m t_f \tag{12}$$

where: t_f - front time, *s* (Fig. 1).

B. The Current Tail

The fundamental requirements for the current tail are:

- to have the maximum steepness at its beginning thus providing a steady transition from one part to the other,
- to reach correct amplitude value.
- to describe the current tail.

A suitable mathematical expression for current tail is given by:

$$I = I_1 e^{-\frac{t-t_n}{t_1}} - I_2 e^{-\frac{t-t_n}{t_2}} \tag{13}$$

where: t_1, t_2 - time constants, s,

I_1, I_2 - constants, A,

t_h - time to half value, s.

The constants are:

$$t_1 = \frac{t_h - t_n}{ln\,2} \tag{14}$$

$$t_2 = \frac{0.1I}{S_m} \tag{15}$$

$$I_1 = \frac{t_1 . t_2}{t_1 - t_2}\left(S_m + 0.9\frac{I}{t_2}\right) \tag{16}$$

$$I_2 = \frac{t_1 . t_2}{t_1 - t_2}\left(S_m + 0.9\frac{I}{t_1}\right) \tag{17}$$

In the publication [3,8] was suggested approximation of the lightning current course by use of a triangular impulse with equivalent time to maximum:

$$t_f = \left(\frac{I_F}{S_m/_{I_F}}\right) \tag{18}$$

where: $S_m/_{I_F}$ conditional distribution of steepness (Table 2)

Table 2 Parameters of first negative lightning downward strokes [3,7,10].

Parameter	Lightning Current Stroke			
	3 kA ≤ I ≤20 kA		I > 20 kA	
	M	β	M	β
I_F - final crest current, kA	61.1	1.330	33.3	0.605
S_m - maximum front steepness, kA μs^{-1}	24.3	0.599	24.3	0.99
$t_m = I_F/S_m$ - minimum equivalent front, s	2.51	1.230	1.37	0.670
S_m/I_F - maximum rate of rise of current conditional distribution, kA μs^{-1}	$12.0\, I_F^{0.171}$	0.554	$6.50\, I_F^{0.376}$	0.554
t_m/I_F, s	$0.0834\, I_F^{0.828}$	0.554	$0.154 I_F$ 0.624	0.554
t_h - wave tail time to half value, μs	77.5	0.557	77.5	0.557
ρ_c - correction factor between t_m and I_F	0.89		0.56	

Results and discussion

I.Simulations of Courses of Lightning Current Strokes

On the basis of the presented model of lightning current stroke courses the selected lightning currents with different maximal values were simulated. Current time to crest t_f was determined by use of equation (18). Figure 6 shows the courses of strokes with maximal value of 100 kA, 33.3 kA, and 10 kA.

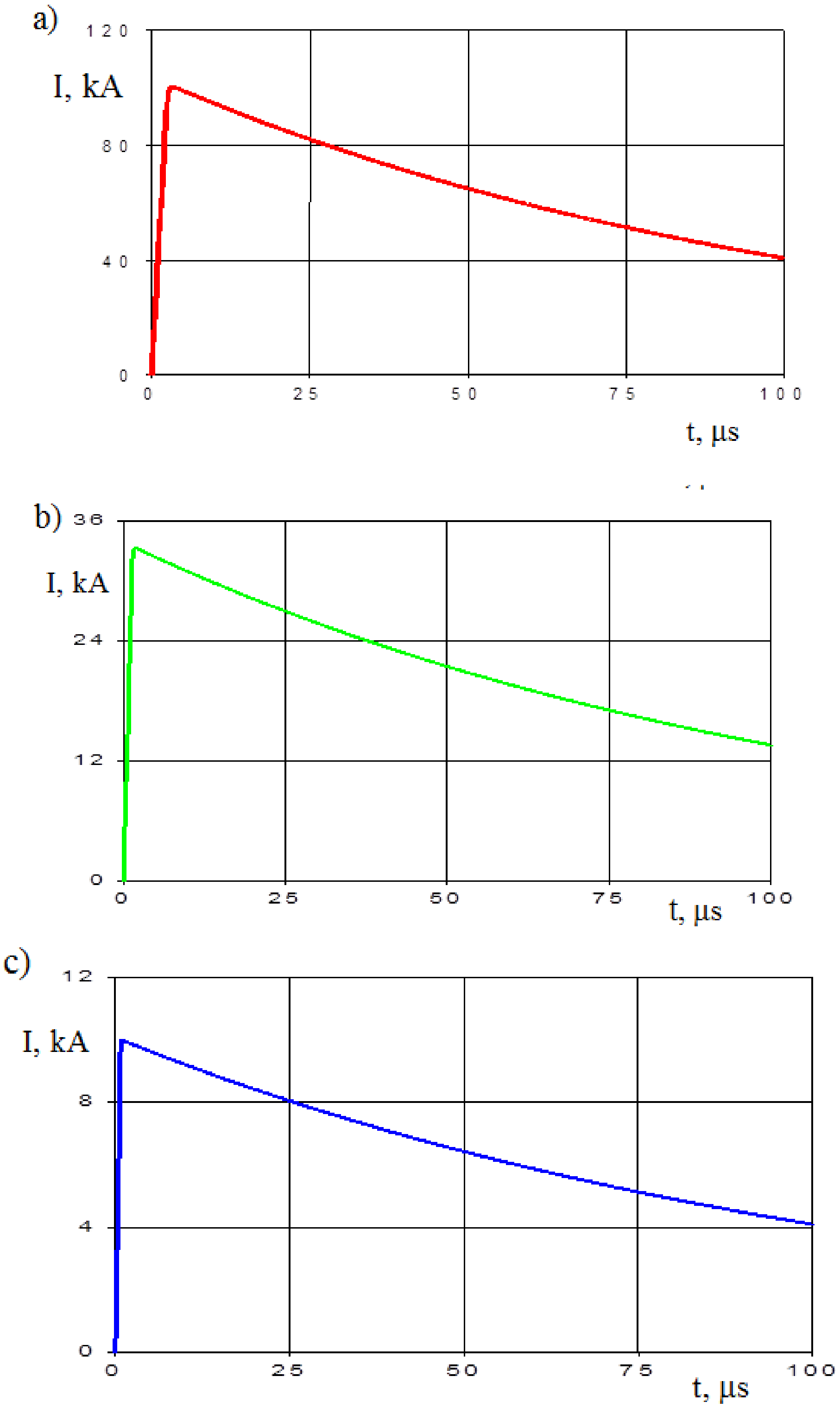

Figure 6. Time courses of selected current strokes determined on base presented model:

a - I_{max} =100 kA (t_f =2,67s), b - I_{max} =33.3 kA (median value) (t_f = 1,88s), c - I_{max}= 10 kA (t_f = 0.56s)

In this analysis, a 220 kV transmission line is used as an object of analysis. For analysis, in the analytical model, a lightning stroke is generated at the center of the transmission line tower. The standard 220 kV transmission line model, shown in Figure 3, are used for analysis. In conducting this analysis, in addition to changing parameters, analysis using EMTP by applying three types of lightning stroke waveforms is undertaken, and the results compared as in Figure 2, first, a ramp wave, with a wave-front time of 2.67 µs, and the amplitude is 100 kA, for 33.3 kA with a wave-front time 1.88 µs, and for 10 kA with a wave front-time of 0.56 µs. The return-stroke current rises to its peak in a few microseconds and slowly decays after reaching the peak, the severity of lightning-caused transient overvoltage's. However, the influence of the peak of the current wave, Ip, and tf, is very significant.

Conclusion

This paper presents an analytical representation of lightning current impulses, emphasizing their waveform characteristics, parameters, and mathematical modeling. The double-exponential function and alternative models pro-vide accurate approximations of real lightning surges. Simulations demonstrate how these models aid in evaluating the effects of lightning on power and structural systems. Future research should focus on improving model accuracy using machine learning and real-time lightning monitoring data.

References

[1] Horvath T. (1991). Computation of Lightning Protection, John Wiley & Sons Inc, New York-Singapore,.

[2] Chowdhuri P., Fellow, IEEE. (2001). Parameters of Lightning Strokes and Their Effects on Power Systems, IEEE/PES, Conf. and Exposition Transmission and Distribution, Atlanta, Georgia, paper Nicole, R. (in press). Title of paper with only first word capitalized. Journal Name Standard Abbreviation.

[3] Motoyama H. (1996). Experimental Study and Analysis of Breakdown Characteristics of Long Air Gaps with Short Tail Lightning Impulse. IEEE Trans. on Pow. Deliv., Vol. 11, No. 2, pp. 972 – 979.

[4] Ihbal, A. M., & Khallifah, A. A. (2023). Forecasting short-term peak load demand in the Libyan power grid using multiple regression model. *Sebha University Journal of Pure & Applied Sciences, 22*(3), 125–128. https://doi.org/10.51984/JOPAS.V22I3.2759.

[5] Golde R. H. (1977). Lightning, Academic Press, London, LTD,.

[6] Rioual M., et. Al. (1989). Short and Long Air Gaps (Insulator Strings and Spark Gaps) Modeling Studies with the EMTP program, EPRI, Palo Alto, California.

[7] Takamitsu I., Toshiaki U., Hideto W., Toshihisa F., Akihiro A. (2003, April). Lightning Flashovers on 77-kV Systems: Observed Voltage Bias Effects and Analysis. IEEE Trans. on Pow. Deliv., Vol. 18, No. 2, pp. 545 – 550.

[8] CIGRE Working Group 33-01 of Study Committee.33. (1991). Overvoltage's and Insulation Coordination: Guide to Procedure for Estimating the Lightning Performance of Transmission Lines. CIGRE Publication, No 63.

[9] CIGRE Working Group 33-01 of Study Committee.33. (1991). Overvoltage's and Insulation Coordination: Guide to Procedure for Estimating the Lightning Performance of Transmission Lines. CIGRE Publication, No 63.

[10] Shindo M., Suzuki T.: A New Calculation Method of Breakdown Voltage-Time Characteristics of Long Air Gaps, IEEE Trans. on PAS, Vol. PAS 104, PAS-98, No. 6, pp. 1909 -1915.

[11] Narita T., Yamada T., Mochizuki A., Zaima E. (1979). Observation of Current Waveshapes of Lightning Strokes on Transmission Towers. IEEE Tran. on Power Delivery, Vol. 15, No. 1, pp. 429-435